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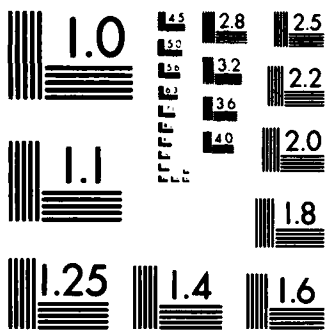

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MEMORANDUM REPORT BRL-MR-3530

STABILITY OF LIQUID-FILLED PROJECTILES WITH UNUSUAL CONING FREQUENCIES

Charles H. Murphy

July 1986

US ARMY BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

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For a liquid-filled shell, a key parameter is τ, the ratio of coning frequency to spin frequency. This report considers the stability implications of τ in the "unusual" intervals (-1,0) and (1,3). It is shown that for a low viscosity liquid treated by the Stewartson-Wedemeyer theory there is no possibility of unstable motion in these two regions.

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I. INTRODUCTION

The prediction of the complete moment exerted by a spinning liquid payload on a spinning and coning projectile has been a problem of considerable interest to the Army for some time. For fully spinning liquid, the linear side moment was first computed by Stewartson[1] for an inviscid payload by use of fluid oscillation eigenfrequencies determined by the fineness ratio of the cylindrical container. Wedemeyer[2] introduced boundary layers on the walls of the container and was able to determine viscous corrections for Stewartson's eigenfrequencies, which could then be used in Stewartson's side moment calculation. Murphy[3] then completed the linear boundary theory by including all pressure and wall shear contributions to the liquid-induced side moment. The Stewartson-Wedemeyer eigenvalue calculations have been improved for low Reynolds numbers by Kitchens *et al*[4] through the replacement of the cylindrical wall boundary approximation by a linearized Navier-Stokes approach. Next, Gerber *et al*[5-6] extended this linearized NS technique to compute better side moment coefficients for Reynolds numbers less than 10,000. Finally, the roll moment for a fully spun-up liquid was computed by Murphy.[7-8]

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1. *Stewartson, K., "On the Stability of a Spinning Top Containing Liquid," Journal of Fluid Mechanics, Vol. 5, Part 4, September 1959, pp. 577-592.*

2. *Wedemeyer, E. H., "Viscous Correction to Stewartson's Stability Criterion," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Report No. 1325, June 1966. (AD 489687)*

3. *Murphy, C. H., "Angular Motion of a Spinning Projectile with a Viscous Liquid Payload," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Memorandum Report ARBRL-MR-03194, August 1982. (AD A118676). (See also Journal of Guidance, Control, and Dynamics, Vol. 6, July-August 1983, pp. 280-286.)*

4. *Kitchens, C. W., Jr., Gerber, N., and Sedney, R., "Oscillations of a Liquid in a Rotating Cylinder: Solid Body Rotation," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Technical Report BRL-TR-02081, June 1978. (AD A057759)*

5. *Gerber, N., Sedney, R., and Bartos, J. M., "Pressure Measurement on a Liquid-Filled Projectile: Solid Body Rotation," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Technical Report ARBRL-TR-02422, October 1982. (AD A120567)*

6. *Gerber, N. and Sedney, R., "Moment on a Liquid-Filled Spinning and Nutating Projectile: Solid Body Rotation," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Technical Report ARBRL-TR-02470, February 1983. (AD A125332)*

7. *Murphy, C. H., "Liquid Payload Roll Moment Induced by a Spinning and Coning Projectile," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Technical Report ARBRL-TR-02521, September 1983. (AD A133681) (See also AIAA Paper 83-2142, August 1983.)*

8. *Murphy, C. H., "A Relation Between Liquid Roll Moment and Liquid Side Moment," Journal of Guidance and Control, Vol. 8, pp. 287-288, March-April 1985.*

In 1965 Greenspan[9] published a general boundary layer theory which for a cylinder should give the same viscous correction to the eigenvalues as computed by Wedemeyer. Kudlick[10] made such calculations but his work was marred by an incorrectly derived formula based on Greenspan's correct general result.* Kudlick also considered only half the axial eigenfunctions and neglected those which are important for coning motion of a projectile.

A primary parameter of the Stewartson-Wedemeyer (SW) theory is τ, the ratio of the coning frequency to the spin frequency. For conventional spinning projectiles, τ lies between zero and one-half. For this reason, all theoretical and experimental work has concentrated on this range of τ. D'Amico[11] considered the possibility of unusual projectiles with τ greater than one and computed Stewartson eigenfrequency tables for $1.1 < \tau < 1.7$. Spinning finned missiles can have coning motions in an opposite direction to the spin and for these missiles τ can be negative.

It is the purpose of this report to consider the stability implication of the SW theory for negative τ and $\tau > 1$. This will be done by first deriving the SW prediction for liquid side moment and then inserting this moment in the projectile dynamics to obtain specific conclusions on the infl .nce of the liquid payload on projectile stability.

II. INVISCID LIQUID MOMENT

We will consider a spinning projectile** performing a coning or spiraling motion. In nonrolling coordinates the angular motion has the form:

$$\tilde{\beta} + i\,\tilde{\alpha} = \hat{K}e^{s\phi} \qquad (2.1)$$

where $\phi = \dot{\phi}t$

$s = (\varepsilon + i)\tau$

$\dot{\phi}$ is the spin rate.

In Appendix A, correct formulas are derived in Kudlick's notation.

***For simplicity only positive spin will be considered.*

9. *Greenspan, H. P., "On the General Theory of Contained Rotating Fluid Motions," Journal of Fluid Mechanics, Vol. 22, Part 3, pp. 449-462, 1965.*

10. *Kudlick, M. D., On the Transient Motions of a Contained, Rotating Fluid, Ph.D Thesis, Department of Mathematics, Massachusetts Institute of Technology, February 1966.*

11. *D'Amico, W. P., "Stability of Satellites and Unusual Aerodynamic-Shaped-Vehicles Carrying Liquid Payloads," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Memorandum Report ARBRL-MR-2392, June 1974. (AD 921454L)*

According to Eq. (2.1), τ is the ratio of the coning frequency to the spin frequency. If the projectile is coning in the direction of spin, τ is positive and positive ϵ indicates a growing motion. If the coning motion opposes the spin, τ is negative and positive ϵ indicates a decaying motion.

The liquid moment induced by the coning motion of Eq. (2.1) can be expressed in terms of a dimensionless liquid moment coefficient.

$$M_{L\tilde{y}} + iM_{L\tilde{z}} = m_L a^2 \dot{\phi}^2 \tau C_{LM} \hat{K} e^{s\phi} \tag{2.2}$$

where $C_{LM} = C_{LSM} + iC_{LIM}$ and other symbols are defined in the List of Symbols. The imaginary part of this coefficient induces a motion in the plane of the angle of attack and is called the liquid in-plane moment coefficient while the real part causes a motion out of this plane and is called the liquid side moment coefficient.

The inviscid liquid moment can be computed from appropriate expressions in Reference 3. Consider a fully filled cylinder with diameter 2a and height 2c, whose center is located at the projectile's center of mass. If we neglect a small centripetal pressure term, the liquid moment coefficient is

$$\begin{aligned} \tau C_{LM} = i(a/2c\hat{K}) \{ & a^{-2}\int_{-c}^{c} [e^{i\phi_p} C_p]_{r=a} x \, dx \\ & -2a^{-3}\int_{0}^{a} [e^{i\phi_p} C_p]_{x=c} r^2 dr \} \end{aligned} \tag{2.3}$$

where $e^{i\phi_p} C_p = -(c/a)\hat{K} \sum a_k E_k \sin(\frac{k\pi x}{2c}) J_1(k\hat{\lambda} r/c)$

$$\hat{\lambda}^2 = -[\frac{s^2-2is+3}{(s-i)^2}] (\pi/2)^2$$

$$a_k = 8(k\pi)^{-2}(-1)^{(k-1)/2} \qquad k = 1, 3, 5, \ldots$$

$$E_k = 2s(s-3i)g^{-1}$$

$$g(s,f^*) = (\frac{s+i}{s-i})J_1(\hat{\lambda}/f^*) - (\hat{\lambda}/f^*)J_0(\hat{\lambda}/f^*)$$

$$f^* = c/ka$$

The k-th term in the series expansion of the liquid moment coefficient can be computed to be:

$$\tau C_{LM_k} = -i(c/a)^2 a_k^2 E_k \left[\frac{s}{s+1} J_1(\hat{\lambda}/f^*) + \frac{(s-i)^2}{2(s^2-2is+3)} g\right] \tag{2.4}$$

For an infinite set, $i\tau_{kn}$, of pure imaginary values of s, the denominator of E_k becomes zero and an infinite value of the inviscid liquid moment is predicted. These eigenfrequencies, τ_{kn}, are functions of the reduced fineness ratio, f^*, and have been tabulated by Stewartson[1] and others.[11-13] It can be shown that the eigenfrequencies lie between -1 and 3. The function g can be expanded in the vicinity of an eigenfrequency.

$$g(s,f^*) = (s-i\tau_{kn})g_1 \tag{2.5}$$

where $g_1 = \left[\frac{\partial}{\partial s} g(s,f^*)\right]_{s=i\tau_{kn}}$

$$= \frac{-2i}{(1-\tau_{kn})^3}\left[(1+\tau_{kn}) + \frac{\pi^2}{2(f^*)^2}\right]\left[J_1(\hat{\lambda}/f^*)\right]_{s=i\tau_{kn}}$$

In the vicinity of τ_{kn}, E_k can be approximated by

$$E_k = \frac{2\tau_{kn}(3-\tau_{kn})}{(s-i\tau_{kn})g_1} \tag{2.6}$$

The liquid moment coefficient for τ's which are not near a τ_{kn} is usually quite small. For τ near a τ_{kn}, it is well approximated by*

**The formula in Reference 3 differs from this by a typographical error. An updated errata for Reference 3 is given in Appendix B.*

12. *Karpov, B. G., "Dynamics of Liquid Filled Shell. Aids for Designers: (a) Milne's Graph; (b) Stewartson's Tables," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Memorandum Report ARBRL-MR-1477, May 1963. (AD 410484)*

13. *Frasier, J. T., "Dynamics of Liquid-Filled Shell: Aids for Designers," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Memorandum Report ARBRL-MR-1892, December 1967. (AD 665356)*

$$\tau C_{LM} = \frac{(f^*)^2 R^2}{2\pi k^2(s - i\tau_{kn})}, \quad -1 < \tau_{kn} < 1$$

$$= \frac{-(f^*)^{-2} R^2}{2\pi k^2(s - i\tau_{kn})}, \quad 1 < \tau_{kn} < 3 \tag{2.7}$$

where $R^2 = \dfrac{4(2f^*)^6|1-\tau_{kn}|^3\tau_{kn}^2(3-\tau_{kn})}{\pi^3(1+\tau_{kn})[\pi^2+2(f^*)^2(1+\tau_{kn})]}$

Values of R are also tabulated as functions of f* in References 1, 11, 12, and 13.

III. VISCOUS MODIFICATIONS

The major difficulty with the inviscid Stewartson eigenfrequency theory is that it predicts much too large liquid moments for τ near an eigenfrequency, τ_{kn}. Wedemeyer[2] introduced a viscous boundary layer theory and sought to predict the resulting liquid moment by a simple manipulation of Stewartson's tables. By a very clever approach, he showed that viscous eigenvalues, s_{kn}, could be computed from Stewartson's table of inviscid eigenfrequencies, τ_{kno}, by the relations

$$s_{kn} = i\tau_{kn} + \varepsilon_{kn}\tau_{kn} \tag{3.1}$$

$$\tau_{kn} = \tau_{kno} + \Delta\tau_{kn} \tag{3.2}$$

$$\varepsilon_{kn}\tau_{kn} + i\Delta\tau_{kn} = i\,\frac{d\tau_{kn}}{df^*}\,f^*\,[\delta_a - \delta_c]_{s\,=\,i\tau_{kn}} \tag{3.3}$$

$$\delta_a = (s - i)^{-1/2}Re^{-1/2}$$

$$\delta_c = \frac{-(a/c)Re^{-1/2}}{2(1+is)}\left[(1 - is)(s - 3i)^{-1/2} - (3 + is)(s + i)^{-1/2}\right]$$

and the complex roots are selected to have positive real parts.

The derivative in Eq. (3.3) can be computed by differentiating the condition for eigenfrequencies, $(g(s,f^*)=0)$.

$$\frac{d\tau_{kn}}{df^*} = -\left[\frac{\frac{\partial g}{\partial f^*}}{i\frac{\partial g}{\partial s}}\right]_{s=i\tau_{kn}} \tag{3.4}$$

But $\left[\frac{\partial g}{\partial s}\right]_{s=i\tau_{kn}} = g_1$

$$\left[\frac{\partial g}{\partial f^*}\right]_{s=i\tau_{kn}} = \frac{-1}{f^*}\,\frac{(1+\tau_{kn})(3-\tau_{kn})}{(1-\tau_{kn})^2}\left[1 + \frac{\pi^2}{4(f^*)^2}\right] J_1\left(\frac{\hat{\lambda}}{f^*}\right)_{s=i\tau_{kn}} \tag{3.6}$$

$$\therefore\; f^*\,\frac{\partial\tau}{\partial f^*} = \frac{(1-\tau_{kn})^2(3-\tau_{kn})[1 + \frac{\pi^2}{4(f^*)^2}]}{[2(1+\tau_{kn}) + \frac{\pi^2}{(f^*)^2}]} \tag{3.7}$$

It can easily be verified that the viscous damping rate, $\varepsilon_{kn}\tau_{kn}$, is always negative. Finally it is interesting to note that if Greenspan's boundary layer theory is correctly applied to a cylinder, viscous corrections to the inviscid eigenfrequency can be computed which agree with those predicted by Eq. (3.3). (See Appendix A.)

The viscous eigenvalue, s_{kn}, of Eq. (3.1) can now be used to replace the inviscid eigenvalue, $i\tau_{kn}$, in Eq. (2.7). The resulting prediction of the liquid moment has agreed very well with experiments for large Reynolds numbers.

IV. LIQUID-FILLED SHELL STABILITY

The angular motion of a spinning projectile can be represented as the sum of two coning motions. In Reference 3 the influence of the liquid moment is considered and it is shown that C_{LIM} affects the frequencies of the motion while C_{LSM} affects the damping. The precise relations are:[3]

$$\tau_j = \sigma/2[f_j - (-1)^j \sqrt{f_j^2 - (1/s_g)}] \qquad j=1,2 \tag{4.1}$$

$$\varepsilon_j = \varepsilon_{aj} + (m_L\; a^2/I_x)[2\tau_j/\sigma - 1]^{-1}\, C_{LSM}(\tau_j,\varepsilon_j) \tag{4.2}$$

where $\sigma = I_x/I_y$

$f_j = 1 + (m_L a^2/I_x) C_{LIM}(\tau_j, \varepsilon_j)$

ε_{aj} is aerodynamic damping rate.

Equation (4.2) can be used to determine the effect of the algebraic sign of C_{LSM} on the stability of the motion. This effect can now be summarized:

(1) $\tau < 0$, positive C_{LSM} is destabilizing.

(2) $0 < \tau < 1$, positive C_{LSM} is destabilizing if $2\tau > \sigma$ and negative C_{LSM} is destabilizing if $2\tau < \sigma$.

(3) $1 < \tau$, positive C_{LSM} is destabilizing*.

For constant amplitude motion ($\varepsilon = 0$), the Stewartson-Wedemeyer theory (Eqs. (2.6, 3.1)) predicts the side moment coefficient for τ near an eigenfrequency to be:

$$C_{LSM} = \frac{-(f^*)^{-2} R^2 \varepsilon_{kn}}{2\pi k^2 [(\tau-\tau_{kn})^2 + \tau_{kn}^2 \varepsilon_{kn}^2]}, \quad -1 < \tau_{kn} < 1$$

$$= \frac{(f^*)^{-2} R^2 \varepsilon_{kn}}{2\pi k^2 [(\tau-\tau_{kn})^2 + \tau_{kn}^2 \varepsilon_{kn}^2]}, \quad 1 < \tau_{kn} < 3 \qquad (4.3)$$

where $\varepsilon_{kn} > 0 \qquad \tau_{kn} < 0$

$\varepsilon_{kn} < 0 \qquad \tau_{kn} > 0.$

According to Equation (4.3), the side moment coefficient is positive for $0 < \tau_{kn} < 1$ and is negative otherwise. Therefore, the Stewartson-Wedemeyer theory predicts a possible instability for the fast rate when τ lies in the range zero to one and no possibility of instability for any other frequencies. In other words, a low viscosity liquid can only have adverse stability effects on conventional statically unstable projectiles. The effect of very viscous liquids can not be predicted by Stewartson-Wedemeyer theory and may be adverse for the stability of any projectile.

**This depends on the very reasonable assumption[11] that $\sigma < 2$.*

REFERENCES

1. Stewartson, K., "On the Stability of a Spinning Top Containing Liquid," Journal of Fluid Mechanics, Vol. 5, Part 4, September 1959, pp. 577-592.

2. Wedemeyer, E. H., "Viscous Correction to Stewartson's Stability Criterion," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Report No. 1325, June 1966. (AD 489687)

3. Murphy, C. H., "Angular Motion of a Spinning Projectile with a Viscous Liquid Payload," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Memorandum Report ARBRL-MR-03194, August 1982. (AD A118676). (See also Journal of Guidance, Control, and Dynamics, Vol. 6, July-August 1983, pp. 280-286.)

4. Kitchens, C. W., Jr., Gerber, N., and Sedney, R., "Oscillations of a Liquid in a Rotating Cylinder: Solid Body Rotation," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Technical Report BRL-TR-02081, June 1978. (AD A057759)

5. Gerber, N., Sedney, R., and Bartos, J. M., "Pressure Measurement on a Liquid-Filled Projectile: Solid Body Rotation," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Technical Report ARBRL-TR-02422, October 1982. (AD A120567)

6. Gerber, N. and Sedney, R., "Moment on a Liquid-Filled Spinning and Nutating Projectile: Solid Body Rotation," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Technical Report ARBRL-TR-02470, February 1983. (AD A125332)

7. Murphy, C. H., "Liquid Payload Roll Moment Induced by a Spinning and Coning Projectile," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Technical Report ARBRL-TR-02521, September 1983. (AD A133681) (See also AIAA Paper 83-2142, August 1983.)

8. Murphy, C. H., "A Relation Between Liquid Roll Moment and Liquid Side Moment," Journal of Guidance and Control, Vol. 8, pp. 287-288, March-April 1985.

9. Greenspan, H. P., "On the General Theory of Contained Rotating Fluid Motions," Journal of Fluid Mechanics, Vol. 22, Part 3, pp. 449-462, 1965.

10. Kudlick, M. D., On the Transient Motions of a Contained, Rotating Fluid, Ph.D Thesis, Department of Mathematics, Massachusetts Institute of Technology, February 1966.

11. D'Amico, W. P., "Stability of Satellites and Unusual Aerodynamic-Shaped-Vehicles Carrying Liquid Payloads," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Memorandum Report ARBRL-MR-2392, June 1974. (AD 921454L)

12. Karpov, B. G., "Dynamics of Liquid Filled Shell. Aids for Designers: (a) Milne's Graph; (b) Stewartson's Tables," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Memorandum Report ARBRL-MR-1477, May 1963. (AD 410484)

13. Frasier, J. T., "Dynamics of Liquid-Filled Shell: Aids for Designers," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Memorandum Report ARBRL-MR-1892, December 1967. (AD 665356)

APPENDIX A

COMMENTS ON REFERENCE 10

The basic error in Reference 10 is Equation [A.3]* which was used to simplify Eq. [2.15]. The result, Eq. [2.16] or (4.29) of Reference 9 is incorrect. The derivation of this equation is based on Eq. [A.4] which uses the solution for the boundary layer velocity vector, $\tilde{\vec{V}}_m$, and applies the divergence theorem to this function. For a cylinder the boundary layer associated with flat end-walls and the cylindrical side-walls overlap and the solution for $\tilde{\vec{V}}_m$ is double-valued. The region for which these two values differ significantly from zero, is, however, quite small and the solution is a quite good approximation. For a sphere, only at the center is the function multiple-valued, but for other spheroids it is multiple-valued throughout the interior, as can be seen from the definition of ζ.

Correct calculations of the viscous correction to the eigenvalue, $s_m^{(1)}$, can be made for a cylinder by use of Eq. [2.15] if different definitions of I_1 and I_2 are used.

$$I_1 = -\int_{-1}^{1}\int_0^{2\pi} [\lim_{\substack{Re\to\infty \\ s\to i\lambda_{mk}}} \Phi_m^* \int_0^\infty \hat{n}\ x(\hat{n}\ x\ \nabla)\cdot \tilde{\vec{V}}_m\, d\zeta]_{r=r_o}\, r_o d\omega dz \qquad \text{(A-1)}$$

$$I_2 = -2\int_0^{r_o}\int_0^{2\pi} [\lim_{\substack{Re\to\infty \\ s\to i\lambda_{mk}}} \Phi_m^* \int_0^\infty \hat{n}\ x(\hat{n}\ x\ \nabla)\cdot \tilde{\vec{V}}_m\, d\zeta]_{z=1}\, r d\omega dr \qquad \text{(A-2)}$$

If the unit vectors for Reference 10's cylindrical coordinates are $(\hat{e}_r, \hat{e}_\omega, \hat{k})$, then $\hat{n}$ is $\hat{e}_r$ for Eq. (A-1) and is $\hat{k}$ for Eq. (A-2).

$$\therefore \hat{n}\ x\ (\hat{n}\ x\ \nabla) = -\hat{e}_\omega\, r^{-1}\frac{\partial}{\partial\omega} - \hat{e}_z\frac{\partial}{\partial z}\ , \qquad \hat{n} = \hat{e}_r$$

$$= -\hat{e}_r\frac{\partial}{\partial r} - \hat{e}_\omega\, r^{-1}\frac{\partial}{\partial\omega}\ , \qquad \hat{n} = \hat{k} \qquad \text{(A-3)}$$

Now, $\Phi_m = \Phi_{mk}(r,z)\, e^{ik\omega}$ (A-4)

$$\vec{Q}_m = (U_{mk}(r,z),\ V_{mk}(r,z),\ W_{mk}(r,z))e^{ik\omega} \qquad \text{(A-5)}$$

* [ ] will be used to denote equations in Reference 10.

$$\frac{\partial \hat{e}_r}{\partial \omega} = \hat{e}_\omega, \quad \frac{\partial \hat{e}_\omega}{\partial \omega} = -\hat{e}_r \tag{A-6}$$

Eqs. (A-4) thru (A-6) plus Eq. [A.4] can be used to simplify Eqs. (A-1) and (A-2).

$$I_1 = +2\pi(i\lambda_{mk})^{-1/2} \int_{-1}^{1} \Phi_{mk} \left(ik\, V_{mk} + r_o \frac{\partial W_{mk}}{\partial z}\right)_{r=r_o} dz \tag{A-7}$$

$$\begin{aligned} I_2 = -\, 2\pi \int_0^{r_o} \{ & (2ip_+)^{-1/2} \left[\frac{\partial}{\partial r} (U_{mk}+iV_{mk}) + \frac{k+1}{r} (U_{mk}+iV_{mk}) \right] \\ & + (2ip_-)^{-1/2} \left[\frac{\partial}{\partial r} (U_{mk}-iV_{mk}) - \frac{k-1}{r} (U_{mk}-iV_{mk}) \right] \}_{z=1} \Phi_{mk} r\, dr \end{aligned} \tag{A-8}$$

If Eq. [7.2] for Φ_{mk}, U_{mk}, V_{mk} and W_{mk} and Bessel function relations [C.2, C.5-C.7] are used in conjunction with Eqs. (A-7, A-8), final equations for I_1 and I_2 can be written:

$$I_1 = +iI_3\hat{\delta}_a \tag{A-9}$$

$$I_2 = -iI_3\hat{\delta}_c \tag{A-10}$$

where
$$I_3 = \frac{2\pi[k^2+(m\pi r_o)^2][J_k(\alpha_{mk} r_o)]^2}{r_o\lambda_{mk}} \tag{A-11}$$

$$\hat{\delta}_a = \left[1 - \frac{i\lambda_{mk}}{|\lambda_{mk}|}\right] |2\lambda_{mk}|^{-1/2} \tag{A-12}$$

$$\begin{aligned} \hat{\delta}_c &= \frac{r_o(1+i)}{4\lambda_{mk}} \left[\frac{(2+\lambda_{mk})}{|p_-|^{1/2}} + \frac{i(2-\lambda_{mk})}{|p_+|^{1/2}} \right] \\ p\pm &= \frac{\lambda_{mk}}{2} \pm 1 \end{aligned} \tag{A-13}$$

N in Reference 10 is defined as

$$N \equiv \iiint |Q|^2 \, dV$$
$$= (4-\lambda_{mk})^{-2} \iiint [|\nabla\Phi_{mk}|^2 + \frac{4}{\lambda_{mk}^2} |k \cdot \nabla\Phi_m|^2] \, dV \qquad \text{(A-14)}$$

It is correctly computed in Eq. [C.16] to be

$$N = 2\pi \left(\frac{J_k(\alpha_{mk} r_o)}{\lambda_{mk}}\right)^2 \frac{4}{4-\lambda_{mk}^2} \{k^2 + (m\pi \ r_o)^2 - \frac{k\lambda_{mk}}{2}\} \qquad \text{(A-15)}$$

$$\therefore s_{mk}^{(1)} = \frac{-(I_1+I_2)}{N}$$
$$= iI_4(\hat{\delta}_a - \hat{\delta}_c) \qquad \text{(A-16)}$$

where

$$I_4 = \frac{-[1 - \frac{\lambda_{mk}^2}{4}]\lambda_{mk} \, [k^2+(m \ \pi r_o)^2]}{r_o \, [k^2+(m \ \pi r_o)^2 - \frac{k\lambda_{mk}}{2}]} \qquad \text{(A-17)}$$

Eqs. [7.7] thru [7.10] and the figures and tables at the end of Chapter 7 need to be revised in the light of the above equations.

A correspondence between the symbols of Reference 10 and that of Reference 3 is given in Table A-1

TABLE A-1

	Reference 10	Reference 3
axial wave number	m	$k/2$
azimuthal wave number	k	$-m$
radial wave number	ℓ	n
inviscid eigenfrequency	$\lambda_{mk\ell}$	$\tau_{knm} - m$
viscous correction to eigenfrequency	$s_{mk\ell}^{(1)}$	$(c/a)[\varepsilon_{knm}\tau_{knm} + i\Delta\tau_{knm}]Re^{1/2}$

reduced fineness ratio	$(2mr_0)^{-1}$	f^*
Reynolds number	R	$(c/a)^2$ Re
sidewall boundary layer parameter	$\hat{\delta}_a$	$\delta_a \, Re^{-1/2}$
endwall boundary layer parameter	$\hat{\delta}_c$	$\delta_c \, Re^{-1/2}$

As can be seen from the table, Kudlick's selection of integer values of his axial wave number, m, considers half of the possible eigenvalues,i.e., only those corresponding to the even values of Stewartson's wave number. For the forced oscillation of a cylindrical payload, k = -1 and m equal to odd multiples of 1/2 provide the significant eigenvalues.

The relations of Table A-1 can be used to express I_4 in terms of the symbols of Reference 3. If this is done, it can be seen that I_4 is $(a/c)f^* \dfrac{\partial \tau}{\partial f^*}$ and, therefore, Eq. (A-17) gives essentially the same viscous corrections as Eq. (3.3).

APPENDIX B

ERRATA FOR BRL MR 3194 (REFERENCE 3)

1. Third term of Eq. (5.20) should contain (s - i), not (s - 1).

2. Second line of Eq. (6.4) should read:

$$= i\ (c/2a) \int_{-1}^{1} \hat{x}\ [C_p^*]_{r=a}\ \hat{K}^{-1}\ d\hat{x} + (h/c)^2\ m_{p\ell h} \tag{6.4}$$

3. Two lines after Eq. (6.4) should read

$$m_{p\ell h} = -i(c^3/2a^2h) \int_{-1}^{1} C_{po}(a,\hat{x})\ d\hat{x}\ ; \ldots$$

4. Eq. (6.12) should read:

$$\begin{aligned} m_p &= \frac{[R/f^*]^2}{2\pi k^2(s-i\tau_{kn})} \qquad \tau_{kn} < 1 \\ &= \frac{-[R/f^*]^2}{2\pi k^2(s-i\tau_{kn})} \qquad \tau_{kn} > 1 \end{aligned} \tag{6.12}$$

where $R = R(f^*,f)$

5. Equation at top of page 32 should read:

$$\epsilon_{kn}\tau_{kn} + i\Delta\tau_{kn} = i[\frac{\partial\tau_{kn}}{\partial f^*}(\delta_a - \delta_c)f^* + 2\frac{\partial\tau_{kn}}{\partial f}(f-1)\delta_a]$$

6. Eq. (6.15) should read:

$$(C_{LSM})_{max} = \frac{-(R/f^*)^2}{2\pi k^2\tau_{kn}^2[\epsilon_{kn}-\epsilon]}\ , \quad \tau_{kn} < 1 \tag{6.15}$$

7. Third line after Eq. (6.15) should read: "somewhat more complicated since δ_a is independent of k and δ_c varies as k^{-1} for constant f*."

8. Fifth line after Eq. (6.15) should read: "and k^{-1}."

9. In the second line from bottom of page 32 replace "1 to 9." by "3 to 9."

10. In the third line from top of page 33 replace "1 to 25." by "5 to 25."

11. Eq. (7.1) should read:

$$m_{v\ell} = i\,(2\pi\,\hat{K}\,Re)^{-1}\,e^{-s\phi}\int_{-1}^{1}\int_{0}^{2\pi} e^{i\theta}\,m^{*}_{v\ell}\,d\theta\,d\hat{x} \tag{7.1}$$

12. Eq. (7.4) should read

$$m_{ve} = (2a\,\hat{K}\,Re)^{-1}\int_{b}^{a}\left[\hat{x}\,\frac{\partial}{\partial x}\,(w_{sv} - iv_{sv})\right]_{\hat{x}=-1}^{\hat{x}=1} rdr + (h/c)^{2}\,m_{veh} \tag{7.4}$$

where

$$m_{veh} = (2\,\hat{K}\,Re)^{-1}\,(c/ah)\int_{b}^{a}\left[\,\frac{\partial}{\partial x}\,(w_{sv} - iv_{sv})\,\right]_{\hat{x}=-1}^{\hat{x}=1} rdr.$$

13. Third term of Eq. (9.4) should be:

$$(s - i)\,\delta_{a}(ad)\,R''_{k}(d)$$

14. Second line after Eq. (9.8) should read:

$$m_{prh} = i(dc^{3}/2a^{3}h)\int_{-1}^{1} e^{i\theta_{p}}\,C_{po}\,d\hat{x}$$

15. Eq. (10.1) should read:

$$(c/a)_{av} = (c/2)\int_{-1}^{1}\frac{d\hat{x}}{a(\hat{x})}$$

16. Eq. (B23) should have + sign in front of u_{si}.

17. On Page 105 last line; replace "aerodynamic" with "liquid."

LIST OF SYMBOLS

a — radius of a right-circular cylindrical cavity containing liquid

a_k — $8(k\pi)^{-2}(-1)^{(k-1)/2}$, $k=1,3,5,\ldots$

c — half-length of a right-circular cylindrical cavity containing liquid

f^* — c/ka

f_j — $1 + (m_L a^2/I_x)\, C_{LIM}\,(\tau_j,\epsilon_j)$, $j=1,2$

g — $\left(\frac{s+i}{s-i}\right) J_1\left(\frac{\hat{\lambda}}{f^*}\right) - \left(\frac{\hat{\lambda}}{f^*}\right) J_0\left(\frac{\hat{\lambda}}{f^*}\right)$

g_1 — $\left.\frac{\partial g(s,f^*)}{\partial s}\right|_{s=i\tau_{nk}}$

k — azimuthal wave number

m_L — mass of the liquid in the cylindrical cavity

n — radial wave number

r — radial coordinate

s — $(\epsilon+i)\tau$

s_g — gyroscopic stability factor

s_{kn} — eigenvalue of s for the liquid's (k,n)th wave mode

t — time

x — coordinate along the missile's axis of symmetry, positive forward

C_{LM} — $\frac{M_{L\tilde{y}} + i\, M_{L\tilde{z}}}{m_L a^2 \dot{\phi}^2 \tau \hat{K} e^{s\phi}}$

C_{LIM} — liquid in-plane moment coefficient; the imaginary part of C_{LM}

C_{LSM} — liquid side moment coefficient; the real part of C_{LM}

C_p — nondimensional magnitude of the complex pressure coefficient, Eq. (2.3)

E_k — $2s(s-3i)/g$

I_x, I_y	axial and transverse moments of inertia of the projectile.
J_0, J_1	Bessel function of the first kind, of order 0 or 1
$\hat{K}$	initial value of the complex yaw
$M_{L\tilde{y}}, M_{L\tilde{z}}$	transverse components of the liquid moment in an aeroballistic system
NS	Navier-Stokes
R	parameter associated with each eigenfrequency, Eq. (2.7)
Re	Reynolds number
SW	Stewartson-Wedemeyer
$\tilde{\alpha}$	angle of attack in an aeroballistic system
$\tilde{\beta}$	angle of side-slip in an aeroballistic system
δ_a	$(s-i)^{-1/2}\ Re^{-1/2}$
δ_c	$\frac{-(a/c)Re^{-1/2}}{2(1+is)}\left[\frac{1-is}{(s-3i)^{1/2}} - \frac{3+is}{(s+i)^{1/2}}\right]$
ε	damping rate per cycle of the coning motion
ε_{aj}	aerodynamic damping rate of the j-th arm, j=1,2
ε_j	damping rate of the j-th arm, j=1,2
ε_{kn}	damping rate for the liquid's (k,n)th wave mode
$\hat{\lambda}$	$\frac{[(3+is)(1-is)]^{1/2}}{1+is}\left(\frac{\pi}{2}\right)$
σ	I_x/I_y
τ	coning rate/$\dot{\phi}$; nondimensional coning frequency
τ_j	coning frequency for the j-th arm, j=1,2
τ_{kn}	eigenfrequency for the liquid's (k,n)th wave mode

LIST OF SYMBOLS (Cont'd)

ϕ	$\dot{\phi}t$
$\dot{\phi}$	spin rate
ϕ_p	orientation angle of the complex pressure coefficient, Eq. (2.3)

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